Chapter 2: The blackbody spectrum and the “ultraviolet catastrophe”

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Quantum Physics

Thermal radiation

What is thermal radiation?

Thermal radiation is the electromagnetic radiation emitted by a body as a result of its temperature.

- All bodies emit such radiation to their surroundings and absorb such radiation from them.
- Usually, most of the radiation is emitted in frequencies outside the visible range. (for example, at the infrared at room temperature)
- All bodies (solids and liquids) emit a continuous spectrum of radiation.
  1. Practically independent of composition
  2. Strongly dependent on the temperature.

Blackbody Radiation

- When radiation impinges on a body, partly is absorbed and partly is reflected
- A black body is the one that **absorbs all the radiation coming on it**
- Independently of their composition, all blackbodies at the same temperature emit thermal radiation with the same spectrum.
- Examples of black bodies:
  - Body painted in black (reflecting very little light)
  - Cavity connected by a small hole to the outside

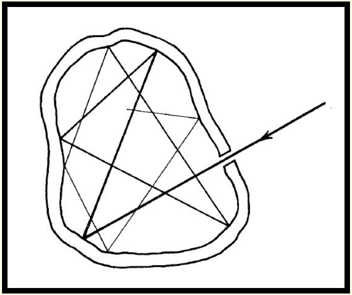


Figure 1. Incident radiation is completely adsorbed after succesive reflections. The radiation emitted by the hole will have a blackbody spectrum

Backbody radiation: experimental results

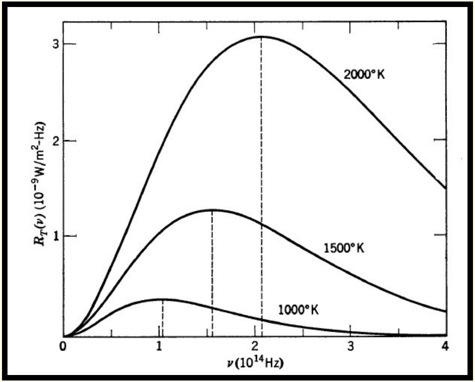


Figure 2. First accurate measurements of $\mathrm{R_T}(\nu)$ by Lummer and Pringsheim (1899)

How do we measure the blackbody spectrum?

We define the **spectral radiancy** $\mathrm{R_T}(\nu)$ such as $\mathrm{R_T}(\nu)\ d\nu$ is the energy emitted per unit time in the frequency interval $[\nu, \nu + d\nu]$ from a unit area of the surface at temperature T.

Total Radiancy

The total energy emitted per unit time per unit area is called the **Radiancy** R_T

$$R_T = \int_0^\infty R_T(\nu)\ d\nu$$

Stefan's and Wien's laws

Stefan's law for the total radiancy (1879)

Figure 2 shows that the total radiancy emitted by a black body increases very rapidly with temperature. It 1879 the following empirical equation was found:

$$R_{\mathrm{T}} = \sigma \mathrm{T}^4 \tag{1}$$

where $\sigma = 5,67 \cdot 10^{-8} W/m^2 K^4$ is called the **Stefan-Boltzmann constant**.

Wien's displacement law (1893)

Figure 2 also shows that the maximum of the spectrum shifts to larger frequencies as T increases, in a linear fashion. This fact is called the Wien's displacement law, first stated in 1893:

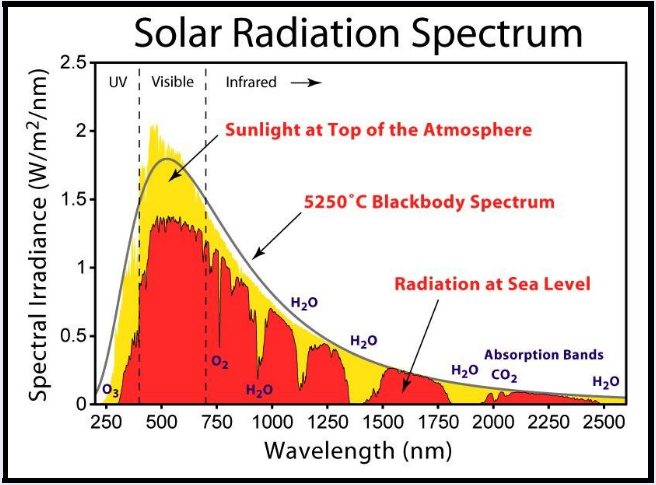
$$\nu_{max} \propto T \tag{2}$$

Then, not only the amount of thermal radiation will increase with temperature, but also the color of a glowing hot body will change, from red to blue-white.
Wien's law can also be put in the form:

$$\lambda_{max} T = 2{,}898 \cdot 10^{-3} m\, K \tag{3}$$

Stars as black bodies

Does the sun behave like a black body?



Conclusion: Wien's law can be used to estimate temperature of stars

Classical theory of cavity radiation

We will now take as a blackbody example the cavity shown in Figure 1 and calculate, using classical physics, the energy density $\rho_T(\nu)$ inside. This quantity is defined as the energy contained in a unit volume of the cavity at temperature T in the frequency interval ν to $\nu + d\nu$, and is related to the spectral radiancy by the relationship

$$R_T(\nu) = \frac{c}{4}\rho_T(\nu) \qquad (4)$$

Light waves

Let's remember that light is classicaly assumed to be electromagnetic waves that propagate in vacuum according to the wave equation for their electric and magnetic components:

$$\frac{\partial^2 E(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E(x,t)}{\partial t^2} = 0 \quad \frac{\partial^2 B(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 B(x,t)}{\partial t^2} = 0 \quad (5)$$

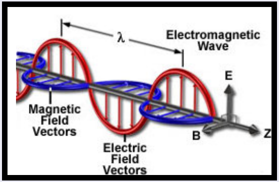
being c the speed of propagation of light ($3 \cdot 10^8$ m/s in vacuum).

Classical theory of cavity radiation

Harmonic waves

The most simple example of an electromagnetic wave is the harmonic or sinusoidal wave. For propagation along the z direction, the solution of the wave equation takes the form:

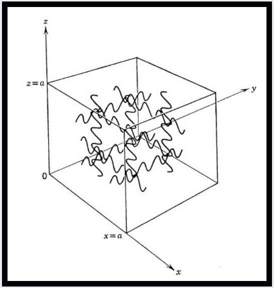
$$\vec{E}(z,t) = E_x \sin(kz - \omega t)\,\vec{i} \qquad \vec{B}(z,t) = B_y \sin(kz - \omega t)\,\vec{j} \qquad (6)$$



k: wave number ω: angular frequency
begin the frequency ν defined as
$\nu = \omega/2\pi = 1/T$ (T = period)
and the wavelenght λ defined as $\lambda = 2\pi/k$
The velocity of propagation c is then
$c = \omega/k = \lambda\nu$
which in vacuum is independent of k

Classical theory of cavity radiation

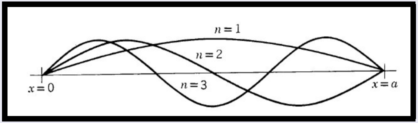
Rayleigh-Jeans calculations



- For simplicity, we assume a metallic cubic cavity filled with electromagnetic radiation. The incident and reflected waves combine to form standing waves.
- As the electric field vector $\vec{\mathbf{E}}$ is parallel to the walls, the standing waves must have nodes at $x = 0$ and $x = a$.
- The electric field for the standing waves is described by

  $E(x, t) = E_0 \sin(2\pi x/\lambda) \sin(2\pi \nu t)$

- Therefore, the waves will have nodes at $2x/\lambda = n \quad (n = 0, 1, 2, ...)$
- At $x = a$, it has to be verified: $2a/\lambda = n \quad (n = 1, 2, ...)$
- This determines a set of allowed values for the wavelength λ

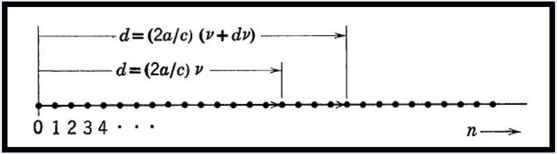


Classical theory of cavity radiation

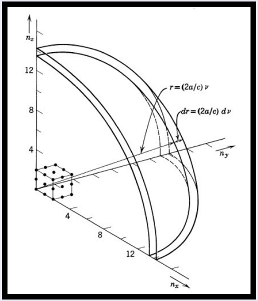
Rayleigh-Jeans calculations

- Working in terms of frequency instead of wavelengths we have: $\nu = cn/2a \quad (n = 1, 2, ...)$
- We now consider, using the diagram below, the number of allowed frequencies in the interval $[\nu, \nu + d\nu]$, or $N(\nu)\,d\nu$.
- Taking into account that we must apply a factor of two to count the two independent polarization states for each wave, we have:

$$N(\nu)\,d\nu = \frac{4a}{c}\,d\nu \tag{7}$$



Rayleigh-Jeans calculations: 3D case



- For the 3D case, we follow the same procedure, counting the number of points within a shell of surface $\pi(2a/c)^2\nu^2$ and thickness $(2a/c)d\nu$
- After working out few mathematical detail (check Eisberg's book), we arrive to:

$$N(\nu)\, d\nu = \frac{8\pi V}{c^3}\, \nu^2\, d\nu \tag{8}$$

being $V = a^3$ the volume of the cavity.

Classical theory of cavity radiation

Rayleigh-Jeans calculations: classical kinetic theory

- The final stage will be to evaluate the average energy contained on each standing wave of frequency ν.
- Applying classical statistical physics, for a system with a large number of physical entities in thermal equilibrium, the law of equipartition of energy applies.
- The average *kinetic* energy per degree of freedom is then $kT/2$, with the Boltzmann constant k being $k = 1{,}38 \cdot 10^{-23} J/K$.
- For a standing electromagnetic wave, the total energy is twice the kinetic energy. Then, we have an average energy per wave $\bar{\epsilon} = kT$, and we can now finally evaluate the energy density inside the cavity $\rho_{\mathrm{T}}(\nu)$ as:

$$\rho_{\mathrm{T}}(\nu)\, d\nu = \frac{N(\nu)\,\bar{\epsilon}}{V}\, d\nu = \frac{8\pi\nu^2 kT}{c^3}\, d\nu \tag{9}$$

  which is the **Rayleigh-Jeans formula for blackbody radiation**.
- At high frequencies, the formula diverges, which constitutes the "ultraviolet catastrophe". To overcome this, we will explain in the next chapter how, by changing the assumptions in classical physics about the energy content of standing waves, Planck arrives to a correct solution of the problem.

Ultraviolet catastrophe